Primary Years Programme

# Mathematics scope and sequence

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**Primary Years Programme**
**Mathematics scope and sequence**

Published February 2009
Updated December 2018

Published on behalf of the International Baccalaureate Organization, a not-for-profit educational foundation of 15 Route des Morillons, 1218 Le Grand-Saconnex, Geneva, Switzerland by the

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Website: www.ibo.org



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To this end the organization works with schools, governments and international organizations to develop challenging programmes of international education and rigorous assessment.

These programmes encourage students across the world to become active, compassionate and lifelong learners who understand that other people, with their differences, can also be right.


# IB learner profile

**The aim of all IB programmes is to develop internationally minded people who, recognizing their common humanity and shared guardianship of the planet, help to create a better and more peaceful world.**

## As IB learners we strive to be:

### INQUIRERS
We nurture our curiosity, developing skills for inquiry and research. We know how to learn independently and with others. We learn with enthusiasm and sustain our love of learning throughout life.

### KNOWLEDGEABLE
We develop and use conceptual understanding, exploring knowledge across a range of disciplines. We engage with issues and ideas that have local and global significance.

### THINKERS
We use critical and creative thinking skills to analyse and take responsible action on complex problems. We exercise initiative in making reasoned, ethical decisions.

### COMMUNICATORS
We express ourselves confidently and creatively in more than one language and in many ways. We collaborate effectively, listening carefully to the perspectives of other individuals and groups.

### PRINCIPLED
We act with integrity and honesty, with a strong sense of fairness and justice, and with respect for the dignity and rights of people everywhere. We take responsibility for our actions and their consequences.

### OPEN-MINDED
We critically appreciate our own cultures and personal histories, as well as the values and traditions of others. We seek and evaluate a range of points of view, and we are willing to grow from the experience.

### CARING
We show empathy, compassion and respect. We have a commitment to service, and we act to make a positive difference in the lives of others and in the world around us.

### RISK-TAKERS
We approach uncertainty with forethought and determination; we work independently and cooperatively to explore new ideas and innovative strategies. We are resourceful and resilient in the face of challenges and change.

### BALANCED
We understand the importance of balancing different aspects of our lives—intellectual, physical, and emotional—to achieve well-being for ourselves and others. We recognize our interdependence with other people and with the world in which we live.

### REFLECTIVE
We thoughtfully consider the world and our own ideas and experience. We work to understand our strengths and weaknesses in order to support our learning and personal development.

**The IB learner profile represents 10 attributes valued by IB World Schools. We believe these attributes, and others like them, can help individuals and groups become responsible members of local, national and global communities.**

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# Mathematics in the Primary Years Programme

## Beliefs and values in mathematics

> All students deserve an opportunity to understand the power and beauty of mathematics.
>
> *Principles and standards for school mathematics*
> National Council of Teachers of Mathematics (NCTM 2000)

In the PYP, mathematics is viewed primarily as a vehicle to support inquiry, providing a global language through which we make sense of the world around us. It is intended that students become competent users of the language of mathematics, and can begin to use it as a way of thinking, as opposed to seeing it as a series of facts and equations to be memorized. The power of mathematics for describing and analysing the world around us is such that it has become a highly effective tool for solving problems.

It is also recognized that students can appreciate the intrinsic fascination of mathematics and explore the world through its unique perceptions. In the same way that students describe themselves as "authors" or "artists", a school's programme should also provide students with the opportunity to see themselves as "mathematicians", where they enjoy and are enthusiastic when exploring and learning about mathematics.

The IB learner profile is integral to learning and teaching mathematics in the PYP because it represents the qualities of effective learners and internationally minded students. The learner profile, together with the other elements of the programme—knowledge, concepts, skills and action—informs planning and teaching in mathematics.

## Effective mathematics practice

It is important that learners acquire mathematical understanding by constructing their own meaning through ever-increasing levels of abstraction, starting with exploring their own personal experiences, understandings and knowledge. Additionally, it is fundamental to the philosophy of the PYP that, since it is to be used in real-life situations, mathematics needs to be taught in relevant, realistic contexts, rather than by attempting to impart a fixed body of knowledge directly to students. How children learn mathematics can be described using the following stages (see figure 1).

**How children learn mathematics**

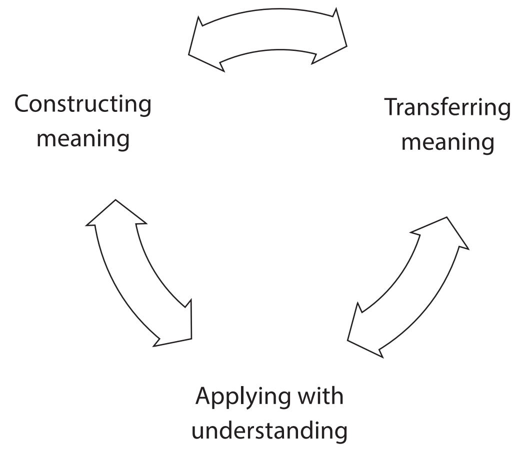


**Figure 1**

It is useful to consider these stages when planning developmentally appropriate learning experiences at all ages. Schools that have local and/or national curriculum requirements in mathematics should articulate how best these can be incorporated into their planning and teaching of mathematics.

## Constructing meaning about mathematics

Learners construct meaning based on their previous experiences and understanding, and by reflecting upon their interactions with objects and ideas. Therefore, involving learners in an active learning process, where they are provided with possibilities to interact with manipulatives and to engage in conversations with others, is paramount to this stage of learning mathematics.

When making sense of new ideas all learners either interpret these ideas to conform to their present understanding or they generate a new understanding that accounts for what they perceive to be occurring. This construct will continue to evolve as learners experience new situations and ideas, have an opportunity to reflect on their understandings and make connections about their learning.

## Transferring meaning into symbols

Only when learners have constructed their ideas about a mathematical concept should they attempt to transfer this understanding into symbols. Symbolic notation can take the form of pictures, diagrams, modelling with concrete objects and mathematical notation. Learners should be given the opportunity to describe their understanding using their own method of symbolic notation, then learning to transfer them into conventional mathematical notation.

## Applying with understanding

Applying with understanding can be viewed as the learners demonstrating and acting on their understanding. Through authentic activities, learners should independently select and use appropriate symbolic notation to process and record their thinking. These authentic activities should include a range of practical hands-on problem-solving activities and realistic situations that provide the opportunity to demonstrate mathematical thinking through presented or recorded formats. In this way, learners are able to apply their understanding of mathematical concepts as well as utilize mathematical skills and knowledge.

As they work through these stages of learning, students and teachers use certain processes of mathematical reasoning.

- They use patterns and relationships to analyse the problem situations upon which they are working.
- They make and evaluate their own and each other's ideas.
- They use models, facts, properties and relationships to explain their thinking.
- They justify their answers and the processes by which they arrive at solutions.

In this way, students validate the meaning they construct from their experiences with mathematical situations. By explaining their ideas, theories and results, both orally and in writing, they invite constructive feedback and also lay out alternative models of thinking for the class. Consequently, all benefit from this interactive process.

Play and exploration have a vital role in the learning and application of mathematical knowledge, particularly for younger students. In a PYP learning environment, mathematics skills and activities need to occur in authentic settings. As educators, we need to provide a variety of areas and resources to allow students to encounter situations that will introduce and develop these skills. In this environment, students will be actively involved in a range of activities that can be free or directed. In planning the learning environment and experiences, teachers need to consider that young students may need to revisit areas and skills many times before understanding can be reached. Applying mathematical skills to real-world tasks supports students' learning.

A PYP teacher's personal knowledge of mathematics is of key importance. What teachers themselves understand shapes which resources they choose, what learning experiences they design and how effectively they teach. The teacher's own interest in, and development of, the subject is maintained through regular professional development, reading of professional journals and, especially, through regular contact with colleagues who share their commitment to teaching mathematics through inquiry. Commercially available resources for teaching mathematics are carefully evaluated to ensure that they meet the needs of the teacher and the students, and the requirements of the programme.

Students and teachers should use the key concepts and related questions (presented later in this section) to guide their inquiries. Teachers should regard these as prompts for developing suitable activities to address the mathematics skills and concepts required.

## The role of mathematics in the programme of inquiry

Wherever possible, mathematics should be taught through the relevant, realistic context of the units of inquiry. The direct teaching of mathematics in a unit of inquiry may not always be feasible but, where appropriate, introductory or follow-up activities may be useful to help students make connections between the different aspects of the curriculum. Students also need opportunities to identify and reflect on "big ideas" within and between the different strands of mathematics, the programme of inquiry and other subjects.

Links to the transdisciplinary themes should be explicitly made, whether or not the mathematics is being taught within the programme of inquiry. A developing understanding of these links will contribute to the students' understanding of mathematics in the world and to their understanding of the transdisciplinary theme. The role of inquiry in mathematics is important, regardless of whether it is being taught inside or outside the programme of inquiry. However, it should also be recognized that there are occasions when it is preferable for students to be given a series of strategies for learning mathematical skills in order to progress in their mathematical understanding rather than struggling to proceed.

# How mathematics practices are changing

Guided inquiry is the main approach to learning and teaching mathematics in the PYP. The PYP represents an approach to teaching that is broad and inclusive in that it provides a context within which a wide variety of teaching strategies and styles can be accommodated, provided that they are driven by a spirit of inquiry and a clear sense of purpose.

The following set of subject-specific examples of effective practice has been produced. It is believed that these examples are worthy of consideration by anyone committed to continuous improvement.

| How are mathematics practices changing? | |
|---|---|
| **Increased emphasis on:** | **Decreased emphasis on:** |
| connecting mathematical concepts and applications to learning | treating mathematics as isolated concepts and facts |
| manipulatives, to make mathematics understandable to students | rote learning, memorization and symbol manipulation |
| real-life problem solving using mathematics | word problems as problem solving |
| instruction built on what students know, what they want to know, and how they best might find out | instruction focused on what students do not know |
| a variety of strategies for possible multiple solutions—emphasis on process | one answer, one method, emphasis on answer |
| students being encouraged to speculate and pursue hunches | the teacher as the sole authority for right answers |
| a broad range of topics regardless of computational skills | computational mastery before moving on to other topics |
| mathematics as a means to an end | teaching mathematics disconnected from other learning |
| the use of calculators and computers for appropriate purposes | a primary emphasis on pencil and paper computations |
| programme of inquiry as the context for learning | the textbook as the context for learning |
| students investigating, questioning, discussing, justifying and journalling their mathematics | the use of worksheets |
| students and teachers engaged in mathematical discourse. | teacher telling about mathematics. |

# Knowledge and skills in mathematics

The mathematics component of the curriculum of the PYP encompasses measurement, shape and number, and their many applications to students' everyday lives. Mathematics provides opportunities for students to engage in investigations into measurement, shape and number, and allows them to communicate in a language that is concise and unambiguous. Mathematics concepts and skills can also be applied to solve a variety of real-life problems. Students apply their mathematical reasoning to a number of situations in order to find an appropriate answer to the problems they wish to solve.

In the PYP, the mathematics component of the curriculum should be driven by concepts and skills rather than by content. The key concepts are inevitably influential in driving the curriculum, but there are many other related mathematics concepts that provide further understanding of the subject.

*Mathematics scope and sequence* identifies the expectations considered appropriate in the PYP. Within each of these interconnected strands, there should be a balance between the acquisition of knowledge and skills and the development of conceptual understanding. The mathematics knowledge component is arranged into five strands: **data handling, measurement, shape and space, pattern and function** and **number**.

In the pattern and function and number strands, students and teachers inquire into number systems and their operations, patterns and functions. They become fluent users of the language of mathematics as they learn to understand its meanings, symbols and conventions.

Data handling, measurement and shape and space are the areas of mathematics that other disciplines use to research, describe, represent and understand aspects of their domain. Mathematics provides the models, systems and processes for handling data, making and comparing measurements, and solving spatial problems. These three strands are, therefore, best studied in authentic contexts provided by the transdisciplinary units of inquiry.

All curriculum areas provide an opportunity to utilize the approaches to learning. The mathematics component of the curriculum also provides opportunities for students to:

- count, sort, match and compare objects, shapes and numbers
- recognize and continue patterns (and relationships)
- use mathematical vocabulary and symbols (including informal mathematics)
- develop and implement/trial strategies for investigating a range of mathematical questions or problems
- select and use appropriate mathematics (operations, computations and units) to solve numerical and word problems
- make reasonable estimates
- analyse, make predictions and infer from data
- become confident and competent users of technology in mathematics learning.

# Mathematics strands

## What do we want students to know?

**Data handling**

Data handling allows us to make a summary of what we know about the world and to make inferences about what we do not know.

- Data can be collected, organized, represented and summarized in a variety of ways to highlight similarities, differences and trends; the chosen format should illustrate the information without bias or distortion.
- Probability can be expressed qualitatively by using terms such as "unlikely", "certain" or "impossible". It can be expressed quantitatively on a numerical scale.

**Measurement**

To measure is to attach a number to a quantity using a chosen unit. Since the attributes being measured are continuous, ways must be found to deal with quantities that fall between numbers. It is important to know how accurate a measurement needs to be or can ever be.

**Shape and space**

The regions, paths and boundaries of natural space can be described by shape. An understanding of the interrelationships of shape allows us to interpret, understand and appreciate our two-dimensional (2D) and three-dimensional (3D) world.

**Pattern and function**

To identify pattern is to begin to understand how mathematics applies to the world in which we live. The repetitive features of patterns can be identified and described as generalized rules called "functions". This builds a foundation for the later study of algebra.

**Number**

Our number system is a language for describing quantities and the relationships between quantities. For example, the value attributed to a digit depends on its place within a base system.

Numbers are used to interpret information, make decisions and solve problems. For example, the operations of addition, subtraction, multiplication and division are related to one another and are used to process information in order to solve problems. The degree of precision needed in calculating depends on how the result will be used.

**Related concepts:** There are many related concepts that could provide further links to the transdisciplinary programme of inquiry or further understanding of the subject. Related concepts, such as pattern, boundaries and base systems, have been embedded into the descriptions for each of the strands above. Schools may choose to develop further related concepts.

# Key concepts in the PYP: What do we want students to understand about mathematics?

Central to the philosophy of the PYP is the principle that guided inquiry is a powerful vehicle for learning that promotes meaning and understanding, and challenges students to engage with significant ideas. Hence in the PYP there is also a commitment to a **concept-driven curriculum** as a means of supporting that inquiry. There are clusters of ideas that can usefully be grouped under a set of overarching concepts, each of which has major significance within and across subjects, regardless of time or place.

These key concepts are one of the essential elements of the PYP framework. It is accepted that these are not, in any sense, the only concepts worth exploring. Taken together they form a powerful curriculum component that drives the teacher- and/or student-constructed inquiries that lie at the heart of the PYP curriculum.

When viewed as a set of questions, the concepts form a research tool that is manageable, open-ended and more readily accessible to students. It is these questions, used flexibly by teachers and students when planning an inquiry-based unit, that shape that unit, giving it direction and purpose.

The following table explains each concept from both the generic perspective and the mathematics perspective.

| Concept | Generic perspective | Mathematics perspective |
|---|---|---|
| **Form**<br>What is it like? | Everything has a form with recognizable features that can be observed, identified, described and categorized. | The recognition, categorization and description of patterns throughout the curriculum. |
| **Function**<br>How does it work? | Everything has a purpose, a role or a way of behaving that can be investigated. | The examination of systems, relationships, mechanics, components and patterns. |
| **Causation**<br>Why is it like it is? | Things do not just happen. There are causal relationships at work, and actions have consequences. | An examination of the mathematical concepts and processes that influence the way things are. |
| **Change**<br>How is it changing? | Change is the process of movement from one state to another. It is universal and inevitable. | Looking for evidence of change, analysing the evidence, drawing conclusions and making predictions. |
| **Connection**<br>How is it connected to other things? | We live in a world of interacting systems in which the actions of any individual element affect others. | The examination of systems and strategies to identify different kinds and levels of relationships, within and between different strands of mathematics and beyond to other subjects. |

| Concept | Generic perspective | Mathematics perspective |
|---|---|---|
| **Perspective**<br>What are the points of view? | Knowledge is moderated by perspectives. Different perspectives lead to different interpretations, understandings and findings. Perspectives may be individual, group, cultural or disciplinary. | The examination of different ways individuals and cultures use mathematics to solve problems.<br>Developing respect for varied interpretations, explanations, strategies and solutions. |
| **Responsibility**<br>What is our responsibility? | People make choices based on their understandings, and the actions they take as a result do make a difference. | Understanding the importance of communicating accurately and appreciating the obligation to apply mathematics with honesty. |

# Examples of questions that illustrate the key concepts

The following table provides sample teacher/student questions that illustrate the key concepts, and that may help to structure or frame an inquiry. These examples demonstrate broad, open-ended questioning—requiring investigation, discussion, and a full and considered response—that is essential in an inquiry-led programme.

| Concept | Sample teacher/student questions |
|---|---|
| **Form**<br>What is it like? | • What is a pattern?<br>• How can we describe these shapes?<br>• What is a fraction?<br>• How can we describe time? |
| **Function**<br>How does it work? | • How does the scale on a graph work?<br>• What happens if we keep adding?<br>• What is each shape being used for?<br>• How can we record time? |
| **Causation**<br>Why is it like it is? | • Why is a block the best shape for building a tower?<br>• Why do these calculations produce patterns?<br>• What prompted people to develop a place value system?<br>• Why was the data displayed in this form? |
| **Change**<br>How is it changing? | • How can we convert from the 12-hour clock to the 24-hour clock?<br>• How can you change one quadrilateral into another?<br>• What do all patterns have in common?<br>• What would happen to the area of something if … ? |

| Concept | Sample teacher/student questions |
|---|---|
| **Connection**<br>How is it connected to other things? | • How can you use fractions to explain musical notation?<br>• How are 4 + 3 and 3 + 4 connected?<br>• What do you already know that helps you to read and interpret this display of data?<br>• How is area connected to perimeter? |
| **Perspective**<br>What are the points of view? | • Are there some different ways of explaining this?<br>• Who might be interested in, or be able to use, the results of our survey?<br>• How do people calculate in different cultures?<br>• What would make this game fair to all players? |
| **Responsibility**<br>What is our responsibility? | • What makes your answer reasonable?<br>• Why does the measurement need to be accurate?<br>• How have you collected all the relevant data? |

## Overall expectations in mathematics

*Mathematics scope and sequence* has been designed in recognition that learning mathematics is a developmental process and that the phases a learner passes through are not always linear or age related. For this reason the content is presented in continuums for each of the five **strands** of mathematics—data handling, measurement, shape and space, pattern and function, and number. The content of each continuum has been organized into four **phases** of development, with each phase building upon and complementing the previous phase. The continuums make explicit the **conceptual understandings** that need to be developed at each phase. Evidence of these understandings is described in the behaviours or **learning outcomes** associated with each phase and these learning outcomes relate specifically to mathematical concepts, knowledge and skills. Additionally, the learning outcomes have been written to reflect the stages a learner goes through when developing conceptual understanding in mathematics—**constructing meaning, transferring meaning into symbols** and **applying with understanding** (see figure 1).

## The structure of the PYP mathematics scope and sequence

This scope and sequence aims to provide information for the whole school community of the learning that is going on in the subject of mathematics. It has been designed in recognition that learning mathematics is a developmental process and that the phases a learner passes through are not always linear or age related. For this reason the content is presented in continuums for each of the five **strands** of mathematics—data handling, measurement, shape and space, pattern and function, and number. For each of the strands there is a strand description and a set of **overall expectations**. The overall expectations provide a summary of the understandings and subsequent learning being developed for each phase within a strand.

The content of each continuum has been organized into four **phases** of development, with each phase building upon and complementing the previous phase. The continuums make explicit the **conceptual understandings** that need to be developed at each phase. Evidence of these understandings is described in the behaviours or **learning outcomes** associated with each phase and these learning outcomes relate specifically to mathematical concepts, knowledge and skills.

The learning outcomes have been written to reflect the stages a learner goes through when developing conceptual understanding in mathematics—**constructing meaning**, **transferring meaning into symbols** and **applying with understanding** (see figure 1). To begin with, the learning outcomes identified in the constructing meaning stage strongly emphasize the need for students to develop understanding of mathematical concepts in order to provide them with a secure base for further learning. In the planning process, teachers will need to discuss the ways in which students may demonstrate this understanding. The amount of time and experiences dedicated to this stage of learning will vary from student to student.

The learning outcomes in the transferring meaning into symbols stage are more obviously demonstrable and observable. The expectation for students working in this stage is that they have demonstrated understanding of the underlying concepts before being asked to transfer this meaning into symbols. It is acknowledged that, in some strands, symbolic representation will form part of the constructing meaning stage. For example, it is difficult to imagine how a student could construct meaning about the way in which information is expressed as organized and structured data without having the opportunity to collect and represent this data in graphs. In this type of example, perhaps the difference between the two stages is that in the transferring meaning into symbols stage the student will be able to demonstrate increased independence with decreasing amounts of teacher prompting required for them to make connections. Another difference could be that a student's own symbolic representation may be extended to include more conventional methods of symbolic representation.

In the final stage, a number of learning outcomes have been developed to reflect the kind of actions and behaviours that students might demonstrate when applying with understanding. It is important to note that other forms of application might be in evidence in classrooms where there are authentic opportunities for students to make spontaneous connections between the learning that is going on in mathematics and other areas of the curriculum and daily life.

When a continuum for a particular strand is observed as a whole, it is clear how the conceptual understandings and the associated learning outcomes develop in complexity as they are viewed across the phases. In each of the phases, there is also a vertical progression where most learning outcomes identified in the constructing meaning stage of the phase are often described as outcomes relating to the transferring meaning into symbols and applying with understanding stages of the same phase. However, on some occasions, a mathematical concept is introduced in one phase but students are not expected to apply the concept until a later phase. This is a deliberate decision aimed at providing students with adequate time and opportunities for the ongoing development of understanding of particular concepts.

Each of the continuums contains a **notes** section which provides extra information to clarify certain learning outcomes and to support planning, learning and teaching of particular concepts.

The scope and sequence also identifies the overall expectations considered appropriate in the PYP. These overall expectations (outlined here) are not a requirement of the programme. It is recommended that schools undertake a careful consideration of their own scope and sequence document in order to identify the overall expectations in mathematics for their students.

# How to use the PYP mathematics scope and sequence

In the course of reviewing *Mathematics scope and sequence*, a decision was made to provide schools with a view of how students construct meaning about mathematics concepts in a more developmental way rather than in fixed age bands. Teachers will need to be given time to discuss this introduction and accompanying continuums and how they can be used to inform planning, teaching and assessing of mathematics in the school. The following points should also be considered in this discussion process.

- It is acknowledged that there are earlier and later phases that have not been described in these continuums.
- Each learner is a unique individual with different life experiences and no two learning pathways are the same.
- Learners within the same age group will have different proficiency levels and needs; therefore, teachers should consider a range of phases when planning mathematics learning experiences for a class.
- Learners are likely to display understanding and skills from more than one of the phases at a time. Consequently, it is recognized that teachers will interpret this scope and sequence according to the needs of their students and their particular teaching situations.
- The continuums are not prescriptive tools that assume a learner must attain all the outcomes of a particular phase before moving on to the next phase, nor that the learner should be in the same phase for each strand.

Each teacher needs to identify the extent to which these factors affect the learner. Plotting a mathematical profile for each student is a complex process for PYP teachers. Prior knowledge should therefore never be assumed before embarking on the presentation or introduction of a mathematical concept.

Schools may decide to use and adapt the PYP scope and sequences according to their needs. For example, a school may decide to frame their mathematics scope and sequence document around the conceptual understandings outlined in the PYP document, but develop other aspects (for example outcomes, indicators, benchmarks, standards) differently. Alternatively, they may decide to incorporate the continuums from the PYP documents into their existing school documents. Schools need to be mindful of practice C1.23 in the IB *Programme standards and practices* (2005) that states, “If the school adapts, or develops, its own scope and sequence documents for each PYP subject, the level of overall expectation regarding student achievement expressed in these documents at least matches that expressed in the PYP scope and sequence documents”. To arrive at such a judgment, and given that the overall expectations in *Mathematics scope and sequence* are presented as broad generalities, it is recommended that the entire document be read and considered.

# Viewing a unit of inquiry through the lens of mathematics

The following diagram shows a sample process for viewing a unit of inquiry through the lens of mathematics. This has been developed as an example of how teachers can identify the mathematical concepts, skills and knowledge required to successfully engage in the units of inquiry.

**Note:** It is important that the integrity of a central idea and ensuing inquiry is not jeopardized by a subject-specific focus too early in the collaborative planning process. Once an inquiry has been planned through to identification of learning experiences, it would be appropriate to consider the following process.

**Will mathematics inform this unit?**

Do aspects of the **transdisciplinary theme** initially stand out as being mathematics related?

Will mathematical knowledge, concepts and skills be needed to understand the **central idea**?

Will mathematical knowledge, concepts and skills be needed to develop the **lines of inquiry** within the unit?

What mathematical knowledge, concepts and skills will the students need to be able to engage with and/or inquire into the following? (Refer to mathematics scope and sequence documents.)

- Central idea
- Lines of inquiry
- Assessment tasks
- Teacher questions, student questions
- Learning experiences

In your planning team, list the knowledge, concepts and skills.

What prior knowledge, concepts and skills do the students have that can be utilized and built upon?

Which stages of understanding are the learners in the class working at—constructing meaning, transferring meaning into symbols, or applying with understanding?

How will we know what they have learned? Identify opportunities for assessment.

Decide which aspects can be learned:

- within the unit of inquiry (learning through mathematics)
- as subject-specific, prior to being used and applied in the context of the inquiry (inquiry into mathematics).

**Figure 2**

*Sample processes for viewing a unit of inquiry through the lens of mathematics*

# Learning continuums

## Data handling

Data handling allows us to make a summary of what we know about the world and to make inferences about what we do not know.

- Data can be collected, organized, represented and summarized in a variety of ways to highlight similarities, differences and trends; the chosen format should illustrate the information without bias or distortion.
- Probability can be expressed qualitatively by using terms such as "unlikely", "certain" or "impossible". It can be expressed quantitatively on a numerical scale.

### Overall expectations

#### Phase 1

Learners will develop an understanding of how the collection and organization of information helps to make sense of the world. They will sort, describe and label objects by attributes and represent information in graphs including pictographs and tally marks. The learners will discuss chance in daily events.

#### Phase 2

Learners will understand how information can be expressed as organized and structured data and that this can occur in a range of ways. They will collect and represent data in different types of graphs, interpreting the resulting information for the purpose of answering questions. The learners will develop an understanding that some events in daily life are more likely to happen than others and they will identify and describe likelihood using appropriate vocabulary.

#### Phase 3

Learners will continue to collect, organize, display and analyse data, developing an understanding of how different graphs highlight different aspects of data more efficiently. They will understand that scale can represent different quantities in graphs and that mode can be used to summarize a set of data. The learners will make the connection that probability is based on experimental events and can be expressed numerically.

#### Phase 4

Learners will collect, organize and display data for the purposes of valid interpretation and communication. They will be able to use the mode, median, mean and range to summarize a set of data. They will create and manipulate an electronic database for their own purposes, including setting up spreadsheets and using simple formulas to create graphs. Learners will understand that probability can be expressed on a scale (0–1 or 0%–100%) and that the probability of an event can be predicted theoretically.

# Learning continuum for data handling

| Phase 1 | Phase 2 | Phase 3 | Phase 4 |
|---|---|---|---|
| **Conceptual understandings**<br>We collect information to make sense of the world around us.<br>Organizing objects and events helps us to solve problems.<br>Events in daily life involve chance. | **Conceptual understandings**<br>Information can be expressed as organized and structured data.<br>Objects and events can be organized in different ways.<br>Some events in daily life are more likely to happen than others. | **Conceptual understandings**<br>Data can be collected, organized, displayed and analysed in different ways.<br>Different graph forms highlight different aspects of data more efficiently.<br>Probability can be based on experimental events in daily life.<br>Probability can be expressed in numerical notations. | **Conceptual understandings**<br>Data can be presented effectively for valid interpretation and communication.<br>Range, mode, median and mean can be used to analyse statistical data.<br>Probability can be represented on a scale between 0–1 or 0%–100%.<br>The probability of an event can be predicted theoretically. |
| **Learning outcomes**<br>When **constructing meaning** learners:<br>• understand that sets can be organized by different attributes<br>• understand that information about themselves and their surroundings can be obtained in different ways<br>• discuss chance in daily events (impossible, maybe, certain). | **Learning outcomes**<br>When **constructing meaning** learners:<br>• understand that sets can be organized by one or more attributes<br>• understand that information about themselves and their surroundings can be collected and recorded in different ways<br>• understand the concept of chance in daily events (impossible, less likely, maybe, most likely, certain). | **Learning outcomes**<br>When **constructing meaning** learners:<br>• understand that data can be collected, displayed and interpreted using simple graphs, for example, bar graphs, line graphs<br>• understand that scale can represent different quantities in graphs<br>• understand that the mode can be used to summarize a set of data<br>• understand that one of the purposes of a database is to answer questions and solve problems<br>• understand that probability is based on experimental events. | **Learning outcomes**<br>When **constructing meaning** learners:<br>• understand that different types of graphs have special purposes<br>• understand that the mode, median, mean and range can summarize a set of data<br>• understand that probability can be expressed in scale (0–1) or per cent (0%–100%)<br>• understand the difference between experimental and theoretical probability. |
| When **transferring meaning into symbols** learners:<br>• represent information through pictographs and tally marks<br>• sort and label real objects by attributes. | When **transferring meaning into symbols** learners:<br>• collect and represent data in different types of graphs, for example, tally marks, bar graphs | When **transferring meaning into symbols** learners:<br>• collect, display and interpret data using simple graphs, for example, bar graphs, line graphs<br>• identify, read and interpret range and scale on graphs | When **transferring meaning into symbols** learners:<br>• collect, display and interpret data in circle graphs (pie charts) and line graphs<br>• identify, describe and explain the range, mode, median and mean in a set of data |

| Phase 1 | Phase 2 | Phase 3 | Phase 4 |
|---|---|---|---|
| | • represent the relationship between objects in sets using tree, Venn and Carroll diagrams<br>• express the chance of an event happening using words or phrases (impossible, less likely, maybe, most likely, certain). | • identify the mode of a set of data<br>• use tree diagrams to express probability using simple fractions. | • set up a spreadsheet using simple formulas to manipulate data and to create graphs<br>• express probabilities using scale (0–1) or per cent (0%–100%). |
| When **applying with understanding** learners:<br>• create pictographs and tally marks<br>• create living graphs using real objects and people*<br>• describe real objects and events by attributes. | When **applying with understanding** learners:<br>• collect, display and interpret data for the purpose of answering questions<br>• create a pictograph and sample bar graph of real objects and interpret data by comparing quantities (for example, more, fewer, less than, greater than)<br>• use tree, Venn and Carroll diagrams to explore relationships between data<br>• identify and describe chance in daily events (impossible, less likely, maybe, most likely, certain). | When **applying with understanding** learners:<br>• design a survey and systematically collect, organize and display data in pictographs and bar graphs<br>• select appropriate graph form(s) to display data<br>• interpret range and scale on graphs<br>• use probability to determine mathematically fair and unfair games and to explain possible outcomes<br>• express probability using simple fractions. | When **applying with understanding** learners:<br>• design a survey and systematically collect, record, organize and display the data in a bar graph, circle graph, line graph<br>• identify, describe and explain the range, mode, median and mean in a set of data<br>• create and manipulate an electronic database for their own purposes<br>• determine the theoretical probability of an event and explain why it might differ from experimental probability. |
| **Notes**<br>Units of inquiry will be rich in opportunities for collecting and organizing information. It may be useful for the teacher to provide scaffolds, such as questions for exploration, and the modelling of graphs and diagrams.<br>*Living graphs refer to data that is organized by physically moving and arranging students or actual materials in such a way as to show and compare quantities. | **Notes**<br>An increasing number of computer and web-based applications are available that enable learners to manipulate data in order to create graphs.<br>Students should have a lot of experience of organizing data in a variety of ways, and of talking about the advantages and disadvantages of each. Interpretations of data should include the information that cannot be concluded as well as that which can. It is important to remember that the chosen format should illustrate the information without bias. | **Notes**<br>Using data that has been collected and saved is a simple way to begin discussing the mode. A further extension of mode is to formulate theories about why a certain choice is the mode.<br>Students should have the opportunity to use databases, ideally, those created using data collected by the students then entered into a database by the teacher or together. | **Notes**<br>A database is a collection of data, where the data can be displayed in many forms. The data can be changed at any time. A spreadsheet is a type of database where information is set out in a table. Using a common set of data is a good way for students to start to set up their own databases. A unit of inquiry would be an excellent source of common data for student practice. |

| Phase 1 | Phase 2 | Phase 3 | Phase 4 |
|---|---|---|---|
| Very young children view the world as a place of possibilities. The teacher should try to introduce practical examples and should use appropriate vocabulary. Discussions about chance in daily events should be relevant to the context of the learners. | Situations that come up naturally in the classroom, often through literature, present opportunities for discussing probability. Discussions need to take place in which students can share their sense of likelihood in terms that are useful to them. | Situations that come up naturally in the classroom or form part of the units of inquiry present opportunities for students to further develop their understanding of statistics and probability concepts. | Technology gives us the option of creating a graph at the press of a key. Being able to generate different types of graphs allows learners to explore and appreciate the attributes of each type of graph and its efficacy in displaying the data.<br><br>Technology also gives us the possibility of rapidly replicating random events. Computer and web-based applications can be used to toss coins, roll dice, and tabulate and graph the results. |

# Measurement

To measure is to attach a number to a quantity using a chosen unit. Since the attributes being measured are continuous, ways must be found to deal with quantities that fall between numbers. It is important to know how accurate a measurement needs to be or can ever be.

## Overall expectations

### Phase 1

Learners will develop an understanding of how measurement involves the comparison of objects and the ordering and sequencing of events. They will be able to identify, compare and describe attributes of real objects as well as describe and sequence familiar events in their daily routine.

### Phase 2

Learners will understand that standard units allow us to have a common language to measure and describe objects and events, and that while estimation is a strategy that can be applied for approximate measurements, particular tools allow us to measure and describe attributes of objects and events with more accuracy. Learners will develop these understandings in relation to measurement involving length, mass, capacity, money, temperature and time.

### Phase 3

Learners will continue to use standard units to measure objects, in particular developing their understanding of measuring perimeter, area and volume. They will select and use appropriate tools and units of measurement, and will be able to describe measures that fall between two numbers on a scale. The learners will be given the opportunity to construct meaning about the concept of an angle as a measure of rotation.

### Phase 4

Learners will understand that a range of procedures exists to measure different attributes of objects and events, for example, the use of formulas for finding area, perimeter and volume. They will be able to decide on the level of accuracy required for measuring and using decimal and fraction notation when precise measurements are necessary. To demonstrate their understanding of angles as a measure of rotation, the learners will be able to measure and construct angles.

# Learning continuum for measurement

| Phase 1 | Phase 2 | Phase 3 | Phase 4 |
|---|---|---|---|
| **Conceptual understandings**<br>Measurement involves comparing objects and events.<br>Objects have attributes that can be measured using non-standard units.<br>Events can be ordered and sequenced. | **Conceptual understandings**<br>Standard units allow us to have a common language to identify, compare, order and sequence objects and events.<br>We use tools to measure the attributes of objects and events.<br>Estimation allows us to measure with different levels of accuracy. | **Conceptual understandings**<br>Objects and events have attributes that can be measured using appropriate tools.<br>Relationships exist between standard units that measure the same attributes. | **Conceptual understandings**<br>Accuracy of measurements depends on the situation and the precision of the tool.<br>Conversion of units and measurements allows us to make sense of the world we live in.<br>A range of procedures exists to measure different attributes of objects and events. |
| **Learning outcomes**<br>When **constructing meaning** learners:<br>• understand that attributes of real objects can be compared and described, for example, longer, shorter, heavier, empty, full, hotter, colder<br>• understand that events in daily routines can be described and sequenced, for example, before, after, bedtime, storytime, today, tomorrow. | **Learning outcomes**<br>When **constructing meaning** learners:<br>• understand the use of standard units to measure, for example, length, mass, money, time, temperature<br>• understand that tools can be used to measure<br>• understand that calendars can be used to determine the date, and to identify and sequence days of the week and months of the year<br>• understand that time is measured using universal units of measure, for example, years, months, days, hours, minutes and seconds. | **Learning outcomes**<br>When **constructing meaning** learners:<br>• understand the use of standard units to measure perimeter, area and volume<br>• understand that measures can fall between numbers on a measurement scale, for example, 3½ kg, between 4 cm and 5 cm<br>• understand relationships between units, for example, metres, centimetres and millimetres<br>• understand an angle as a measure of rotation. | **Learning outcomes**<br>When **constructing meaning** learners:<br>• understand procedures for finding area, perimeter and volume<br>• understand the relationships between area and perimeter, between area and volume, and between volume and capacity<br>• understand unit conversions within measurement systems (metric or customary). |
| When **transferring meaning into symbols** learners:<br>• identify, compare and describe attributes of real objects, for example, longer, shorter, heavier, empty, full, hotter, colder | When **transferring meaning into symbols** learners:<br>• estimate and measure objects using standard units of measurement: length, mass, capacity, money and temperature | When **transferring meaning into symbols** learners:<br>• estimate and measure using standard units of measurement: perimeter, area and volume | When **transferring meaning into symbols** learners:<br>• develop and describe formulas for finding perimeter, area and volume<br>• use decimal and fraction notation in measurement, for example, 3.2 cm, 1.47 kg, 1½ miles |

| Phase 1 | Phase 2 | Phase 3 | Phase 4 |
| --- | --- | --- | --- |
| • compare the length, mass and capacity of objects using non-standard units<br>• identify, describe and sequence events in their daily routine, for example, before, after, bedtime, storytime, today, tomorrow. | • read and write the time to the hour, half hour and quarter hour<br>• estimate and compare lengths of time: second, minute, hour, day, week and month. | • describe measures that fall between numbers on a scale<br>• read and write digital and analogue time on 12-hour and 24-hour clocks. | • read and interpret scales on a range of measuring instruments<br>• measure and construct angles in degrees using a protractor<br>• carry out simple unit conversions within a system of measurement (metric or customary). |
| When **applying with understanding** learners:<br>• describe observations about events and objects in real-life situations<br>• use non-standard units of measurement to solve problems in real-life situations involving length, mass and capacity. | When **applying with understanding** learners:<br>• use standard units of measurement to solve problems in real-life situations involving length, mass, capacity, money and temperature<br>• use measures of time to assist with problem solving in real-life situations. | When **applying with understanding** learners:<br>• use standard units of measurement to solve problems in real-life situations involving perimeter, area and volume<br>• select appropriate tools and units of measurement<br>• use timelines in units of inquiry and other real-life situations. | • When **applying with understanding** learners:<br>• select and use appropriate units of measurement and tools to solve problems in real-life situations<br>• determine and justify the level of accuracy required to solve real-life problems involving measurement<br>• use decimal and fractional notation in measurement, for example, 3.2 cm, 1.47 kg, 1½ miles<br>• use timetables and schedules (12-hour and 24-hour clocks) in real-life situations<br>• determine times worldwide. |

| Phase 1 | Phase 2 | Phase 3 | Phase 4 |
|---|---|---|---|
| **Notes**<br>Learners need many opportunities to experience and quantify measurement in a direct kinesthetic manner. They will develop understanding of measurement by using manipulatives and materials from their immediate environment, for example, containers of different sizes, sand, water, beads, corks and beans. | **Notes**<br>Using materials from their immediate environment, learners can investigate how units are used for measurement and how measurements vary depending on the unit that is used. Learners will refine their estimation and measurement skills by basing estimations on prior knowledge, measuring the object and comparing actual measurements with their estimations. | **Notes**<br>In order to use measurement more authentically, learners should have the opportunity to measure real objects in real situations. The units of inquiry can often provide these realistic contexts.<br>A wide range of measuring tools should be available to the students, for example, rulers, trundle wheels, tape measures, bathroom scales, kitchen scales, timers, analogue clocks, digital clocks, stopwatches and calendars. There are an increasing number of computer and web-based applications available for students to use in authentic contexts.<br>Please note that outcomes relating to angles also appear in the shape and space strand. | **Notes**<br>Learners generalize their measuring experiences as they devise procedures and formulas for working out perimeter, area and volume.<br>While the emphasis for understanding is on measurement systems commonly used in the learner's world, it is worthwhile being aware of the existence of other systems and how conversions between systems help us to make sense of them. |

# Shape and space

The regions, paths and boundaries of natural space can be described by shape. An understanding of the interrelationships of shape allows us to interpret, understand and appreciate our two-dimensional (2D) and three-dimensional (3D) world.

## Overall expectations

### Phase 1

Learners will understand that shapes have characteristics that can be described and compared. They will understand and use common language to describe paths, regions and boundaries of their immediate environment.

### Phase 2

Learners will continue to work with 2D and 3D shapes, developing the understanding that shapes are classified and named according to their properties. They will understand that examples of symmetry and transformations can be found in their immediate environment. Learners will interpret, create and use simple directions and specific vocabulary to describe paths, regions, positions and boundaries of their immediate environment.

### Phase 3

Learners will sort, describe and model regular and irregular polygons, developing an understanding of their properties. They will be able to describe and model congruency and similarity in 2D shapes. Learners will continue to develop their understanding of symmetry, in particular reflective and rotational symmetry. They will understand how geometric shapes and associated vocabulary are useful for representing and describing objects and events in real-world situations.

### Phase 4

Learners will understand the properties of regular and irregular polyhedra. They will understand the properties of 2D shapes and understand that 2D representations of 3D objects can be used to visualize and solve problems in the real world, for example, through the use of drawing and modelling. Learners will develop their understanding of the use of scale (ratio) to enlarge and reduce shapes. They will apply the language and notation of bearing to describe direction and position.

# Learning continuum for shape and space

| Phase 1 | Phase 2 | Phase 3 | Phase 4 |
| --- | --- | --- | --- |
| **Conceptual understandings**<br>Shapes can be described and organized according to their properties.<br>Objects in our immediate environment have a position in space that can be described according to a point of reference. | **Conceptual understandings**<br>Shapes are classified and named according to their properties.<br>Some shapes are made up of parts that repeat in some way.<br>Specific vocabulary can be used to describe an object's position in space. | **Conceptual understandings**<br>Changing the position of a shape does not alter its properties.<br>Shapes can be transformed in different ways.<br>Geometric shapes and vocabulary are useful for representing and describing objects and events in real-world situations. | **Conceptual understandings**<br>Manipulation of shape and space takes place for a particular purpose.<br>Consolidating what we know of geometric concepts allows us to make sense of and interact with our world.<br>Geometric tools and methods can be used to solve problems relating to shape and space. |
| **Learning outcomes**<br>When **constructing meaning** learners:<br>• understand that 2D and 3D shapes have characteristics that can be described and compared<br>• understand that common language can be used to describe position and direction, for example, inside, outside, above, below, next to, behind, in front of, up, down. | **Learning outcomes**<br>When **constructing meaning** learners:<br>• understand that there are relationships among and between 2D and 3D shapes<br>• understand that 2D and 3D shapes can be created by putting together and/or taking apart other shapes<br>• understand that examples of symmetry and transformations can be found in their immediate environment<br>• understand that geometric shapes are useful for representing real-world situations<br>• understand that directions can be used to describe pathways, regions, positions and boundaries of their immediate environment. | **Learning outcomes**<br>When **constructing meaning** learners:<br>• understand the common language used to describe shapes<br>• understand the properties of regular and irregular polygons<br>• understand congruent or similar shapes<br>• understand that lines and axes of reflective and rotational symmetry assist with the construction of shapes<br>• understand an angle as a measure of rotation<br>• understand that directions for location can be represented by coordinates on a grid<br>• understand that visualization of shape and space is a strategy for solving problems. | **Learning outcomes**<br>When **constructing meaning** learners:<br>• understand the common language used to describe shapes<br>• understand the properties of regular and irregular polyhedra<br>• understand the properties of circles<br>• understand how scale (ratios) is used to enlarge and reduce shapes<br>• understand systems for describing position and direction<br>• understand that 2D representations of 3D objects can be used to visualize and solve problems<br>• understand that geometric ideas and relationships can be used to solve problems in other areas of mathematics and in real life. |

| Phase 1 | Phase 2 | Phase 3 | Phase 4 |
|---|---|---|---|
| When **transferring meaning into symbols** learners:<br>• sort, describe and compare 3D shapes<br>• describe position and direction, for example, inside, outside, above, below, next to, behind, in front of, up, down. | When **transferring meaning into symbols** learners:<br>• sort, describe and label 2D and 3D shapes<br>• analyse and describe the relationships between 2D and 3D shapes<br>• create and describe symmetrical and tessellating patterns<br>• identify lines of reflective symmetry<br>• represent ideas about the real world using geometric vocabulary and symbols, for example, through oral description, drawing, modelling, labelling<br>• interpret and create simple directions, describing paths, regions, positions and boundaries of their immediate environment. | When **transferring meaning into symbols** learners:<br>• sort, describe and model regular and irregular polygons<br>• describe and model congruency and similarity in 2D shapes<br>• analyse angles by comparing and describing rotations: whole turn; half turn; quarter turn; north, south, east and west on a compass<br>• locate features on a grid using coordinates<br>• describe and/or represent mental images of objects, patterns, and paths. | When **transferring meaning into symbols** learners:<br>• analyse, describe, classify and visualize 2D (including circles, triangles and quadrilaterals) and 3D shapes, using geometric vocabulary<br>• describe lines and angles using geometric vocabulary<br>• identify and use scale (ratios) to enlarge and reduce shapes<br>• identify and use the language and notation of bearing to describe direction and position<br>• create and model how a 2D net converts into a 3D shape and vice versa<br>• explore the use of geometric ideas and relationships to solve problems in other areas of mathematics. |
| When **applying with understanding** learners:<br>• explore and describe the paths, regions and boundaries of their immediate environment (inside, outside, above, below) and their position (next to, behind, in front of, up, down). | When **applying with understanding** learners:<br>• analyse and use what they know about 3D shapes to describe and work with 2D shapes<br>• recognize and explain simple symmetrical designs in the environment<br>• apply knowledge of symmetry to problem-solving situations<br>• interpret and use simple directions, describing paths, regions, positions and boundaries of their immediate environment. | When **applying with understanding** learners:<br>• analyse and describe 2D and 3D shapes, including regular and irregular polygons, using geometrical vocabulary<br>• identify, describe and model congruency and similarity in 2D shapes<br>• recognize and explain symmetrical patterns, including tessellation, in the environment<br>• apply knowledge of transformations to problem-solving situations. | When **applying with understanding** learners:<br>• use geometric vocabulary when describing shape and space in mathematical situations and beyond<br>• use scale (ratios) to enlarge and reduce shapes<br>• apply the language and notation of bearing to describe direction and position<br>• use 2D representations of 3D objects to visualize and solve problems, for example using drawings or models. |

| Phase 1 | Phase 2 | Phase 3 | Phase 4 |
|---|---|---|---|
| **Notes**<br>Learners need many opportunities to experience shape and space in a direct kinesthetic manner, for example, through play, construction and movement.<br>The manipulatives that they interact with should include a range of 3D shapes, in particular the real-life objects with which children are familiar. 2D shapes (plane shapes) are a more abstract concept but can be understood as faces of 3D shapes. | **Notes**<br>Learners need to understand the properties of 2D and 3D shapes before the mathematical vocabulary associated with shapes makes sense to them. Through creating and manipulating shapes, learners align their natural vocabulary with more formal mathematical vocabulary and begin to appreciate the need for this precision. | **Notes**<br>Computer and web-based applications can be used to explore shape and space concepts such as symmetry, angles and coordinates.<br>The units of inquiry can provide authentic contexts for developing understanding of concepts relating to location and directions. | **Notes**<br>Tools such as compasses and protractors are commonly used to solve problems in real-life situations. However, care should be taken to ensure that students have a strong understanding of the concepts embedded in the problem to ensure meaningful engagement with the tools and full understanding of the solution. |

# Pattern and function

To identify pattern is to begin to understand how mathematics applies to the world in which we live. The repetitive features of patterns can be identified and described as generalized rules called "functions". This builds a foundation for the later study of algebra.

## Overall expectations

### Phase 1

Learners will understand that patterns and sequences occur in everyday situations. They will be able to identify, describe, extend and create patterns in various ways.

### Phase 2

Learners will understand that whole numbers exhibit patterns and relationships that can be observed and described, and that the patterns can be represented using numbers and other symbols. As a result, learners will understand the inverse relationship between addition and subtraction, and the associative and commutative properties of addition. They will be able to use their understanding of pattern to represent and make sense of real-life situations and, where appropriate, to solve problems involving addition and subtraction.

### Phase 3

Learners will analyse patterns and identify rules for patterns, developing the understanding that functions describe the relationship or rules that uniquely associate members of one set with members of another set. They will understand the inverse relationship between multiplication and division, and the associative and commutative properties of multiplication. They will be able to use their understanding of pattern and function to represent and make sense of real-life situations and, where appropriate, to solve problems involving the four operations.

### Phase 4

Learners will understand that patterns can be represented, analysed and generalized using algebraic expressions, equations or functions. They will use words, tables, graphs and, where possible, symbolic rules to analyse and represent patterns. They will develop an understanding of exponential notation as a way to express repeated products, and of the inverse relationship that exists between exponents and roots. The students will continue to use their understanding of pattern and function to represent and make sense of real-life situations and to solve problems involving the four operations.

## Learning continuum for pattern and function

| Phase 1 | Phase 2 | Phase 3 | Phase 4 |
|---|---|---|---|
| **Conceptual understandings**<br>Patterns and sequences occur in everyday situations.<br><br>Patterns repeat and grow. | **Conceptual understandings**<br>Whole numbers exhibit patterns and relationships that can be observed and described.<br><br>Patterns can be represented using numbers and other symbols. | **Conceptual understandings**<br>Functions are relationships or rules that uniquely associate members of one set with members of another set.<br><br>By analysing patterns and identifying rules for patterns it is possible to make predictions. | **Conceptual understandings**<br>Patterns can often be generalized using algebraic expressions, equations or functions.<br><br>Exponential notation is a powerful way to express repeated products of the same number. |
| **Learning outcomes**<br>When **constructing meaning** learners:<br>• understand that patterns can be found in everyday situations, for example, sounds, actions, objects, nature. | **Learning outcomes**<br>When **constructing meaning** learners:<br>• understand that patterns can be found in numbers, for example, odd and even numbers, skip counting<br>• understand the inverse relationship between addition and subtraction<br>• understand the associative and commutative properties of addition. | **Learning outcomes**<br>When **constructing meaning** learners:<br>• understand that patterns can be analysed and rules identified<br>• understand that multiplication is repeated addition and that division is repeated subtraction<br>• understand the inverse relationship between multiplication and division<br>• understand the associative and commutative properties of multiplication. | **Learning outcomes**<br>When **constructing meaning** learners:<br>• understand that patterns can be generalized by a rule<br>• understand exponents as repeated multiplication<br>• understand the inverse relationship between exponents and roots<br>• understand that patterns can be represented, analysed and generalized using tables, graphs, words, and, when possible, symbolic rules. |
| When **transferring meaning into symbols** learners:<br>• describe patterns in various ways, for example, using words, drawings, symbols, materials, actions, numbers. | When **transferring meaning into symbols** learners:<br>• represent patterns in a variety of ways, for example, using words, drawings, symbols, materials, actions, numbers<br>• describe number patterns, for example, odd and even numbers, skip counting. | When **transferring meaning into symbols** learners:<br>• describe the rule for a pattern in a variety of ways<br>• represent rules for patterns using words, symbols and tables<br>• identify a sequence of operations relating one set of numbers to another set. | When **transferring meaning into symbols** learners:<br>• represent the rule of a pattern by using a function<br>• analyse pattern and function using words, tables and graphs, and, when possible, symbolic rules. |

| Phase 1 | Phase 2 | Phase 3 | Phase 4 |
|---|---|---|---|
| When **applying with understanding** learners:<br>• extend and create patterns. | When **applying with understanding** learners:<br>• extend and create patterns in numbers, for example, odd and even numbers, skip counting<br>• use number patterns to represent and understand real-life situations<br>• use the properties and relationships of addition and subtraction to solve problems. | When **applying with understanding** learners:<br>• select appropriate methods for representing patterns, for example using words, symbols and tables<br>• use number patterns to make predictions and solve problems<br>• use the properties and relationships of the four operations to solve problems. | When **applying with understanding** learners:<br>• select appropriate methods to analyse patterns and identify rules<br>• use functions to solve problems. |
| **Notes**<br>The world is filled with pattern and there will be many opportunities for learners to make this connection across the curriculum.<br>A range of manipulatives can be used to explore patterns including pattern blocks, attribute blocks, colour tiles, calculators, number charts, beans and buttons. | **Notes**<br>Students will apply their understanding of pattern to the numbers they already know. The patterns they find will help to deepen their understanding of a range of number concepts.<br>Four-function calculators can be used to explore number patterns. | **Notes**<br>Patterns are central to the understanding of all concepts in mathematics. They are the basis of how our number system is organized. Searching for, and identifying, patterns helps us to see relationships, make generalizations, and is a powerful strategy for problem solving. Functions develop from the study of patterns and make it possible to predict in mathematics problems. | **Notes**<br>Algebra is a mathematical language using numbers and symbols to express relationships. When the same relationship works with any number, algebra uses letters to represent the generalization. Letters can be used to represent the quantity. |

# Number

Our number system is a language for describing quantities and the relationships between quantities. For example, the value attributed to a digit depends on its place within a base system.

Numbers are used to interpret information, make decisions and solve problems. For example, the operations of addition, subtraction, multiplication and division are related to one another and are used to process information in order to solve problems. The degree of precision needed in calculating depends on how the result will be used.

## Overall expectations

### Phase 1

Learners will understand that numbers are used for many different purposes in the real world. They will develop an understanding of one-to-one correspondence and conservation of number, and be able to count and use number words and numerals to represent quantities.

### Phase 2

Learners will develop their understanding of the base 10 place value system and will model, read, write, estimate, compare and order numbers to hundreds or beyond. They will have automatic recall of addition and subtraction facts and be able to model addition and subtraction of whole numbers using the appropriate mathematical language to describe their mental and written strategies. Learners will have an understanding of fractions as representations of whole-part relationships and will be able to model fractions and use fraction names in real-life situations.

### Phase 3

Learners will develop the understanding that fractions and decimals are ways of representing whole-part relationships and will demonstrate this understanding by modelling equivalent fractions and decimal fractions to hundredths or beyond. They will be able to model, read, write, compare and order fractions, and use them in real-life situations. Learners will have automatic recall of addition, subtraction, multiplication and division facts. They will select, use and describe a range of strategies to solve problems involving addition, subtraction, multiplication and division, using estimation strategies to check the reasonableness of their answers.

### Phase 4

Learners will understand that the base 10 place value system extends infinitely in two directions and will be able to model, compare, read, write and order numbers to millions or beyond, as well as model integers. They will develop an understanding of ratios. They will understand that fractions, decimals and percentages are ways of representing whole-part relationships and will work towards modelling, comparing, reading, writing, ordering and converting fractions, decimals and percentages. They will use mental and written strategies to solve problems involving whole numbers, fractions and decimals in real-life situations, using a range of strategies to evaluate reasonableness of answers.

# Learning continuum for number

| Phase 1 | Phase 2 | Phase 3 | Phase 4 |
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| **Conceptual understandings**<br>Numbers are a naming system.<br>Numbers can be used in many ways for different purposes in the real world.<br>Numbers are connected to each other through a variety of relationships.<br>Making connections between our experiences with number can help us to develop number sense. | **Conceptual understandings**<br>The base 10 place value system is used to represent numbers and number relationships.<br>Fractions are ways of representing whole-part relationships.<br>The operations of addition, subtraction, multiplication and division are related to each other and are used to process information to solve problems.<br>Number operations can be modelled in a variety of ways.<br>There are many mental methods that can be applied for exact and approximate computations. | **Conceptual understandings**<br>The base 10 place value system can be extended to represent magnitude.<br>Fractions and decimals are ways of representing whole-part relationships.<br>The operations of addition, subtraction, multiplication and division are related to each other and are used to process information to solve problems.<br>Even complex operations can be modelled in a variety of ways, for example, an algorithm is a way to represent an operation. | **Conceptual understandings**<br>The base 10 place value system extends infinitely in two directions.<br>Fractions, decimal fractions and percentages are ways of representing whole-part relationships.<br>For fractional and decimal computation, the ideas developed for whole-number computation can apply.<br>Ratios are a comparison of two numbers or quantities. |
| **Learning outcomes**<br>When **constructing meaning** learners:<br>• understand one-to-one correspondence<br>• understand that, for a set of objects, the number name of the last object counted describes the quantity of the whole set<br>• understand that numbers can be constructed in multiple ways, for example, by combining and partitioning<br>• understand conservation of number*<br>• understand the relative magnitude of whole numbers<br>• recognize groups of zero to five objects without counting (subitizing) | **Learning outcomes**<br>When **constructing meaning** learners:<br>• model numbers to hundreds or beyond using the base 10 place value system**<br>• estimate quantities to 100 or beyond<br>• model simple fraction relationships<br>• use the language of addition and subtraction, for example, add, take away, plus, minus, sum, difference<br>• model addition and subtraction of whole numbers<br>• develop strategies for memorizing addition and subtraction number facts<br>• estimate sums and differences | **Learning outcomes**<br>When **constructing meaning** learners:<br>• model numbers to thousands or beyond using the base 10 place value system<br>• model equivalent fractions<br>• use the language of fractions, for example, numerator, denominator<br>• model decimal fractions to hundredths or beyond<br>• model multiplication and division of whole numbers<br>• use the language of multiplication and division, for example, factor, multiple, product, quotient, prime numbers, composite number | **Learning outcomes**<br>When **constructing meaning** learners:<br>• model numbers to millions or beyond using the base 10 place value system<br>• model ratios<br>• model integers in appropriate contexts<br>• model exponents and square roots<br>• model improper fractions and mixed numbers<br>• simplify fractions using manipulatives<br>• model decimal fractions to thousandths or beyond<br>• model percentages<br>• understand the relationship between fractions, decimals and percentages |

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| • understand whole-part relationships<br>• use the language of mathematics to compare quantities, for example, more, less, first, second. | • understand situations that involve multiplication and division<br>• model addition and subtraction of fractions with the same denominator. | • model addition and subtraction of fractions with related denominators***<br>• model addition and subtraction of decimals. | • model addition, subtraction, multiplication and division of fractions<br>• model addition, subtraction, multiplication and division of decimals. |
| When **transferring meaning into symbols** learners:<br>• connect number names and numerals to the quantities they represent. | When **transferring meaning into symbols** learners:<br>• read and write whole numbers up to hundreds or beyond<br>• read, write, compare and order cardinal and ordinal numbers<br>• describe mental and written strategies for adding and subtracting two-digit numbers. | When **transferring meaning into symbols** learners:<br>• read, write, compare and order whole numbers up to thousands or beyond<br>• develop strategies for memorizing addition, subtraction, multiplication and division number facts<br>• read, write, compare and order fractions<br>• read and write equivalent fractions<br>• read, write, compare and order fractions to hundredths or beyond<br>• describe mental and written strategies for multiplication and division. | When **transferring meaning into symbols** learners:<br>• read, write, compare and order whole numbers up to millions or beyond<br>• read and write ratios<br>• read and write integers in appropriate contexts<br>• read and write exponents and square roots<br>• convert improper fractions to mixed numbers and vice versa<br>• simplify fractions in mental and written form<br>• read, write, compare and order decimal fractions to thousandths or beyond<br>• read, write, compare and order percentages<br>• convert between fractions, decimals and percentages. |
| When **applying with understanding** learners:<br>• count to determine the number of objects in a set<br>• use number words and numerals to represent quantities in real-life situations | When **applying with understanding** learners:<br>• use whole numbers up to hundreds or beyond in real-life situations<br>• use cardinal and ordinal numbers in real-life situations | When **applying with understanding** learners:<br>• use whole numbers up to thousands or beyond in real-life situations<br>• use fast recall of multiplication and division number facts in real-life situations | When **applying with understanding** learners:<br>• use whole numbers up to millions or beyond in real-life situations<br>• use ratios in real-life situations<br>• use integers in real-life situations |

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| • use the language of mathematics to compare quantities in real-life situations, for example, more, less, first, second<br>• subitize in real-life situations<br>• use simple fraction names in real-life situations. | • use fast recall of addition and subtraction number facts in real-life situations<br>• use fractions in real-life situations<br>• use mental and written strategies for addition and subtraction of two-digit numbers or beyond in real-life situations<br>• select an appropriate method for solving a problem, for example, mental estimation, mental or written strategies, or by using a calculator<br>• use strategies to evaluate the reasonableness of answers. | • use decimal fractions in real-life situations<br>• use mental and written strategies for multiplication and division in real-life situations<br>• select an efficient method for solving a problem, for example, mental estimation, mental or written strategies, or by using a calculator<br>• use strategies to evaluate the reasonableness of answers<br>• add and subtract fractions with related denominators in real-life situations<br>• add and subtract decimals in real-life situations, including money<br>• estimate sum, difference, product and quotient in real-life situations, including fractions and decimals. | • convert improper fractions to mixed numbers and vice versa in real-life situations<br>• simplify fractions in computation answers<br>• use fractions, decimals and percentages interchangeably in real-life situations<br>• select and use an appropriate sequence of operations to solve word problems<br>• select an efficient method for solving a problem: mental estimation, mental computation, written algorithms, by using a calculator<br>• use strategies to evaluate the reasonableness of answers<br>• use mental and written strategies for adding, subtracting, multiplying and dividing fractions and decimals in real-life situations<br>• estimate and make approximations in real-life situations involving fractions, decimals and percentages. |

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| **Notes**<br>*To conserve, in mathematical terms, means the amount stays the same regardless of the arrangement.<br>Learners who have been encouraged to select their own apparatus and methods, and who become accustomed to discussing and questioning their work, will have confidence in looking for alternative approaches when an initial attempt is unsuccessful.<br>Estimation is a skill that will develop with experience and will help children gain a "feel" for numbers. Children must be given the opportunity to check their estimates so that they are able to further refine and improve their estimation skills.<br>There are many opportunities in the units of inquiry and during the school day for students to practise and apply number concepts authentically. | **Notes**<br>**Modelling involves using concrete materials to represent numbers or number operations, for example, the use of pattern blocks or fraction pieces to represent fractions and the use of base 10 blocks to represent number operations.<br>Students need to use numbers in many situations in order to apply their understanding to new situations. In addition to the units of inquiry, children's literature also provides rich opportunities for developing number concepts.<br>To be useful, addition and subtraction facts need to be recalled automatically. Research clearly indicates that there are more effective ways to do this than "drill and practice". Above all, it helps to have strategies for working them out. Counting on, using doubles and using 10s are good strategies, although learners frequently invent methods that work equally well for themselves.<br>Difficulties with fractions can arise when fractional notation is introduced before students have fully constructed meaning about fraction concepts. | **Notes**<br>Modelling using manipulatives provides a valuable scaffold for constructing meaning about mathematical concepts. There should be regular opportunities for learners to work with a range of manipulatives and to discuss and negotiate their developing understandings with others.<br>***Examples of related denominators include halves, quarters (fourths) and eighths. These can be modelled easily by folding strips or squares of paper.<br>The interpretation and meaning of remainders can cause difficulty for some learners. This is especially true if calculators are being used. For example, $67 \div 4 = 16.75$. This can also be shown as $16\frac{3}{4}$ or 16 r3. Learners need practice in producing appropriate answers when using remainders. For example, for a school trip with 25 students, only buses that carry 20 students are available. A remainder could not be left behind, so another bus would be required!<br>Calculator skills must not be ignored. All answers should be checked for their reasonableness.<br>By reflecting on and recording their findings in mathematics learning logs, students begin to notice patterns in the numbers that will further develop their understanding. | **Notes**<br>It is not practical to continue to develop and use base 10 materials beyond 1,000. Learners should have little difficulty in extending the place value system once they have understood the grouping pattern up to 1,000. There are a number of websites where virtual manipulatives can be utilized for working with larger numbers.<br>Estimation plays a key role in checking the feasibility of answers. The method of multiplying numbers and ignoring the decimal point, then adjusting the answer by counting decimal places, does not give the learner an understanding of why it is done. Application of place value knowledge must precede this application of pattern.<br>Measurement is an excellent way of exploring the use of fractions and decimals and their interchange.<br>Students should be given many opportunities to discover the link between fractions and division.<br>A thorough understanding of multiplication, factors and large numbers is required before working with exponents. |